Information acquisition in conflicts

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ABSTRACT

Information Acquisition in Conflicts

by Florian Morath and Johannes Münster *

This paper considers incentives for information acquisition ahead of conflicts. We characterize the (unique) equilibrium of the all-pay auction between two players with one-sided asymmetric information. The type of one player is common knowledge. The type of the other player is drawn from a continuous distribution and is private information of this player. We then use our results to study information acquisition prior to an all-pay auction. Depending on the cost of information, only one player may invest in information. If the decision to acquire information is observable for the opponent, but not the information received, one-sided asymmetric information can occur endogenously in equilibrium. Moreover, compared with the first best, information acquisition is excessive. In contrast, with open or covert information acquisition, the cut-off values for equilibrium information acquisition are as in the first best.

Keywords: All-pay auctions, conflicts, contests, information acquisition, asymmetric information

JEL Classification: D72, D74, D82, D83

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Informationsbeschaffung in Konflikten

Dieser Aufsatz untersucht Anreize für Informationsbeschaffung im Vorfeld von Konflikten. Zunächst charakterisieren wir das (eindeutige) Gleichgewicht eines vollständig diskriminierenden Wettkampfs (all-pay auction) zwischen zwei Kontrahenten mit einseitig asymmetrischer Information darüber, wie die Kontrahenten den Gewinn bewerten: Die Bewertung eines Spielers ist allgemein bekannt, und die Bewertung des anderen Spielers ist gemäß einer stetigen Verteilungsfunktion verteilt und private Information dieses Spielers. Anschließend verwenden wir unsere Ergebnisse, um Informationsbeschaffung im Vorfeld einer all-pay auction zu untersuchen. Abhängig von den Kosten der Informationsbeschaffung investiert lediglich ein Spieler in Information. Falls die Entscheidung über Informationsbeschaffung für den Gegenspieler beobachtbar ist, jedoch nicht die Information selbst, kann einseitig asymmetrische Information im Gleichgewicht der Entscheidungen über Informationsbeschaffung entstehen. Darüber hinaus investieren die Spieler verglichen mit dem Wohlfahrtsoptimum zu viel in Information. Im Gegensatz dazu ist die Informationsbeschaffung effizient, falls entweder gekaufte Information offen beobachtbar ist oder weder die Information noch die Entscheidungen über deren Beschaffung durch den Kontrahenten beobachtet werden können.

1 Introduction

Contest theory studies the interaction between agents who spend resources in order to increase their chances of winning a prize. A large number of economic environments can fruitfully be analyzed as contests - e.g. advertising of firms, patent races, rent-seeking and lobbying, political campaigning, or litigation. In many of these environments, the competitors do not know exactly what value they would derive from winning, or how costly it is to expend effort. They may, however, be willing to invest a significant amount of time or money in order to find out about the prize that is at stake, or about the cost of competing. Such investments in information have important implications on the interaction in the contest, both on the amount of resources spent and on allocative efficiency. Moreover, as a consequence of information acquisition, contestants may differ in the quality of the information they have about their own or their competitors' valuation.

Asymmetries with regard to the information the contestants possess are a feature of many contests. These asymmetries can arise from decisions on information acquisition prior to the conflict. In other cases, they are features of the environment the contestants compete in. As an example, consider the case of the Brent Spar oil rig that the owners, Royal Dutch Shell and Exxon, wanted to sink in the Atlantic Ocean. Following a worldwide campaign organized by the environmental group Greenpeace, they abandoned this plan and decided to re-use a large part of the rig. This contest was characterized by a one-sided asymmetry of information about the valuations of the contestants. There were publicly accessible estimations of the cost of the on-shore dismantling of Brent Spar, including estimations published by the owners themselves. There was, however, very little public information about the value Greenpeace placed on the prevention of the deep sea disposal so that the owners of Brent Spar had to rely on their guesses about how far Greenpeace would go. Such one-sided asymmetric information is also prominent in many situations where an incumbent competes with a newcomer, for example in regulated markets or in election races.

Private information of the contestants, however, often results from information acquisition. A firm entering a market will try to find out about the market conditions and the potential gains before competing with an incumbent. As another example for investments in information consider again the contest over Brent Spar. One of the first actions of Greenpeace was to enter the Brent Spar and to take some samples of how much toxic material was on it. This clearly allowed Greenpeace to learn more about the value that was at stake.

In this paper, we first study one-sided asymmetric information in a perfectly discriminating contest or all-pay auction between two risk neutral players. The all-pay auction has been used to model a number of contests such as rent-seeking contests and lobbying (Hillman and Riley 1989, Ellingsen 1991, Baye et al. 1993, Polborn 2006), election campaigns (Che and Gale 1998), and also R&D races (Dasgupta 1986); see Konrad (2009) for a recent survey. We characterize the (unique) equilibrium of the all-pay auction between two contestants, where the valuation of one contestant is common knowledge, whereas the valuation of the other contestant is drawn from a continuous distribution and is his private information. In equilibrium, the player whose valuation is commonly known randomizes continuously, whereas the player with private information plays a pure strategy.

We then analyze information acquisition ahead of conflicts and the players' incentives for such investments. Suppose each player initially only knows the distribution of his type, but that he can learn his true valuation by investing some amount. We distinguish between three different cases depending on how much the opponent can observe if a player invests: (i) the opponent can observe whether a player has invested in information, but not the realized valuation of the opponent in case the player invests, (ii) the opponent can observe the outcome of information acquisition (open information acquisition), (iii) the opponent cannot observe at all whether a player has acquired information (covert information acquisition).

In case (i), if no player invests in information, the resulting contest is similar to an all-pay auction with complete information where, by risk neutrality, the benefit of winning is the expected valuation. If both players acquire information, the all-pay auction turns into the well-known framework with private information. If exactly one player invests in information, then the ensuing contest has one-sided asymmetric information: there are common beliefs about the type of one player, while the type of the other player is his private information. In this setting, information acquisition has a strategic effect on the behavior of the opponent in the contest. We show that players are willing to spend a considerable amount on information. Moreover, for intermediate costs of information acquisition, only one player will invest. To be more precise, there are two asymmetric equilibria where exactly one player invests, and there is also a symmetric equilibrium where both players randomize their investment decision. Thus, the case of one-sided asymmetric information can arise endogenously in an equilibrium of the game with information acquisition. Rent dissipation is incomplete, although players are symmetric ex ante. Compared with the first best, information acquisition is excessive in case (i).

In cases (ii) and (iii), the players' equilibrium investments are again guided by cut-off values concerning the cost of information acquisition. We show that, however, these cut-off values are exactly equal to the cut-off values for first best investment in cases (ii) and (iii). Since in case (i), the players' willingness to pay for information is higher, this suggests that there is a strategic value of information acquisition if the players' decisions are observable, but not the information itself.

The paper is related to several studies of the all-pay auction under different assumptions on the information available to the contestants. Hillman and Riley (1989) study the all-pay auction for the two benchmark cases: complete information and private information about the individual valuations. Baye et al. (1996) characterize the set of equilibria of the all-pay auction with N players and complete information. Amann and Leininger (1996) show uniqueness of the equilibrium with two-sided asymmetric information and two ex ante asymmetric players. Morath and Münster (2008) compare private versus complete information in auctions, and find that for the all-pay auction, revenue is smaller under complete information, while bidders' payoffs are the same in the two information structures. Krishna and Morgan (1997) consider the case where the players' signals are affiliated. Konrad (2009) characterizes the equilibrium under one-sided asymmetric information where one player's value follows a two-point distribution. The all-pay auction with multiple prizes is studied by Moldovanu and Sela (2001) in a framework with private information, and by Clark and Riis (1998) and Barut and Kovenock (1998) with complete information.

Closely related to our work are three papers that study one-sided asymmetric information in contests. For a logit contest success function, Hurley and Shogren (1998a) analyze contests with one-sided asymmetric information, and Hurley and Shogren (1998b) compare the three information structures that also arise in our model with regard to rent dissipation and efficiency. For a more general contest success function, Wärneryd (2003) considers an imperfectly discriminating contest with two agents who have the same value of winning, but where there is uncertainty about this value. He compares a symmetric information structure to the case where one agent privately knows the value of the prize and shows that rent dissipation may be lower under asymmetric information. We add to this literature by studying the all-pay auction framework, and we focus on private values.[1]

Our paper is also linked to the literature on strategic behavior ahead of contests. Konrad (2009) surveys this literature. Our contribution to this literature is to study the incentives for information acquisition in contests.

In Section 2, we describe the strategies and payoffs of the players in the all-pay auction for a given information structure. In Section 3, we analyze the all-pay auction with one-sided asymmetric information. In Section 4, we consider the all-pay auction in a context of information acquisition. Section 5 discusses how our result is affected if the assumptions on the observability of information acquisition change. Section 6 is the conclusion. All proofs are in the appendix.

2 The all-pay auction

There are two players 1 and 2 competing in an all-pay auction. Player i values winning by v_i. The *valuations*, or *types*, v_1 and v_2 are drawn independently from a cumulative distribution function F that is common knowledge. F has support $[0, 1]$

[1] One-sided asymmetric information in common value first-price auctions has been studied by Engelbrecht-Wiggans et al. (1983), among others. In addition, a growing literature considers information acquisition in winner-pay auctions; recent work includes Persico (2000) or Hernando-Veciana (2009).

and is continuously differentiable with $F'(v) > 0$ for $v \in (0,1)$.

In Section 3, we assume that the realized value of v_1 is common knowledge, whereas the realized value of v_2 is private information of player 2. In Section 4, we assume that initially no player is informed about any valuation, but players can acquire information: at a cost c, a player can learn his own value.[2] Player j can observe whether or not i has acquired information, but not the realized value v_i.

Finally, players compete in an all-pay auction. They simultaneously choose their bids $x_i \in [0,\infty)$. The player with the higher bid wins, ties are broken randomly. Both players have to pay their bids. Thus, i's payoff from the all-pay auction (gross of the direct cost of investing in information) is

$$u_i = \begin{cases} v_i - x_i, & x_i > x_j, \\ \frac{v_i}{2} - x_i, & x_i = x_j, \\ -x_i, & x_i < x_j. \end{cases}$$

3 One-sided asymmetric information

Suppose that player 1's valuation v_1 is common knowledge.[3] Player 2's valuation v_2 is privately known only to himself. Thus, a pure strategy of player 1 is a bid $x_1 \in [0,\infty)$, whereas a pure strategy of player 2 is a function $\beta_2 : [0,1] \to [0,\infty)$ that maps the typespace into the set of possible bids. The solution concept is Bayesian Nash equilibrium (henceforth, "equilibrium").

Denote the bid distributions of players 1 and 2 by B_1 and B_2, i.e. $B_i(x)$ denotes the probability that i's bid is weakly below x. If 1 plays a pure strategy to bid x with probability one, then B_1 is degenerate: $B_1(z) = 0$ for $z < x$ and $B_1(z) = 1$ otherwise. If B_1 is not degenerate, 1 plays a non-degenerate mixed strategy. In contrast, the bid distribution B_2 captures the uncertainty concerning v_2 as well as the possible randomization of player 2.

[2] Note that the investment does not change the distribution of one's value, nor one's ability to compete in the contest. Investments in one's value or ability have been studied by Münster (2007).

[3] The analysis goes through for all $v_1 > 0$. For $v_1 = 0$, there is no equilibrium because player 1 will bid zero and player 2 has no best response since any strictly positive bid, however small, guarantees victory.

Lemma 1 *In any equilibrium, the bid distributions B_1 and B_2 have the following properties:*

(i) *(Continuity) B_1 and B_2 are continuous on $(0, \infty)$.*

(ii) *(Support) The supports of B_1 and B_2 both have the same minimum $\underline{b}= 0$, and the same maximum $\bar{b} \leq v_1$.*

(iii) *(At most one mass point at zero) $\min \{B_1(0), B_2(0)\} = 0$.*

(iv) *(Monotonicity) B_1 and B_2 are strictly monotone increasing on $\left[0, \bar{b}\right]$.*

Similar properties are standard in auction theory. Continuity implies that there are no interior mass points. Monotonicity rules out any gaps in the support. Thus (ii) and (iv) imply that B_1 and B_2 have the same support.

It follows directly from Lemma 1 that, in any equilibrium, player 1 randomizes according to a CDF that is continuous and strictly increasing on $\left[0, \bar{b}\right]$. To get some intuition, suppose to the contrary that player 1 chooses a pure strategy, i.e. bids some amount x with probability one. Then player 2 would either like to marginally overbid player 1, or bids zero. But then bidding x is not optimal for 1, contradicting equilibrium. Thus player 1 has to randomize. In contrast, 2 plays a pure strategy.

Lemma 2 *In any equilibrium, player 2 plays a pure strategy $\beta_2 : [0, 1] \to \left[0, \bar{b}\right]$. There is a critical value $\underline{v}\in [0, 1)$ such that $\beta_2(v_2) = 0$ for $v_2 \leq\underline{v}$ and $\beta_2(v_2) > 0$ for $v_2 >\underline{v}$. Moreover, β_2 is continuous on $[0, 1]$ and strictly increasing on $[\underline{v}, 1]$.*

Lemma 2 shows that player 2, whose valuation is private information, bids according to a strategy that is increasing in his value, and low types might bid zero. The highest type of player 2 (who has $v_2 = 1$) bids exactly $\bar{b}$. The intuition behind the proof is simple. Higher types of player 2 will bid higher. Thus, if some type of player 2 randomizes over some interval, no other type of player 2 will bid in this interval. But then B_2 is constant in that interval, contradicting Lemma 1.

Note that β_2 has image $\left[0, \bar{b}\right]$. Since β_2 is continuous and strictly increasing on $(\underline{v}, 1]$, it is invertible on $(\underline{v}, 1]$ with $\beta_2^{-1} : \left(0, \bar{b}\right] \to (\underline{v}, 1]$. Furthermore, β_2^{-1} is continuous and strictly increasing on $\left(0, \bar{b}\right]$.

Lemma 3 *In equilibrium,* B_1 *and* B_2 *are differentiable on* $\left(0, \bar{b}\right)$; *moreover* β_2 *is differentiable on* $(\underline{v}, 1)$.

Given differentiability of the bid distributions, we can use first-order conditions to determine the equilibrium and show its uniqueness. The expected payoff of player 1 from a bid $x_1 \in \left(0, \bar{b}\right]$ is equal to

$$E\left[u_1\left(x_1\right)\right] = F\left(\beta_2^{-1}\left(x_1\right)\right) v_1 - x_1$$

since β_2^{-1} exists on $\left(0, \bar{b}\right]$. Because player 1 randomizes continuously on $\left(0, \bar{b}\right]$, $E\left[u_1\left(x_1\right)\right]$ must be constant in this interval. Therefore,

$$F'\left(\beta_2^{-1}\left(x_1\right)\right) \frac{v_1}{\beta_2'\left(\beta_2^{-1}\left(x_1\right)\right)} - 1 = 0. \qquad (1)$$

Any solution to the differential equation (1) has to fulfill

$$\beta_2\left(v_2\right) = F\left(v_2\right) v_1 + k$$

for all v_2 such that $\beta_2\left(v_2\right) > 0$, where the constant k remains to be determined. Note that $F\left(v_2\right) v_1 + k > 0$ if and only if $v_2 > F^{-1}\left(-k/v_1\right)$. By Lemma 2, types $v_2 \leq F^{-1}\left(-k/v_1\right)$ bid zero, hence $B_2(0) = -k/v_1$, and thus $k \in \left(-v_1, 0\right]$. For notational convenience, let $\alpha_2 = -k/v_1$ (we use the subscript '2' since $\alpha_2 = B_2(0)$). Putting things together,

$$\beta_2\left(v_2\right) = \begin{cases} 0, & v_2 \in \left[0, F^{-1}\left(\alpha_2\right)\right) \\ F\left(v_2\right) v_1 - \alpha_2 v_1, & v_2 \in \left[F^{-1}\left(\alpha_2\right), 1\right] \end{cases} \qquad (2)$$

where $\alpha_2 \in [0, 1)$ remains to be determined.

Now consider player 2. The first-order condition for a type v_2 who bids a strictly positive amount is given by

$$B_1'\left(x_2\right) v_2 - 1 = 0. \qquad (3)$$

Using (2),

$$B_1'(x_2) = \frac{1}{\beta_2^{-1}(x_2)} = \frac{1}{F^{-1}\left(\frac{x_2+\alpha_2 v_1}{v_1}\right)} \tag{4}$$

has to hold for all $x_2 > 0$. This is solved by

$$\begin{aligned} B_1(x_2) &= \int_0^{x_2} \frac{1}{F^{-1}\left(\frac{z+\alpha_2 v_1}{v_1}\right)} dz + \alpha_1 \\ &= \int_{F^{-1}(\alpha_2)}^{\beta_2^{-1}(x_2)} \frac{v_1}{v} dF(v) + \alpha_1 \end{aligned} \tag{5}$$

where α_1 remains to be determined. Note that $\alpha_1 = B_1(0) \in [0, 1)$.

To determine α_1 and α_2, we use the fact that, at most, one of the bid distributions has a mass point at zero (Lemma 1(iii)):

$$\min\{B_1(0), B_2(0)\} = \min\{\alpha_1, \alpha_2\} = 0. \tag{6}$$

Moreover, player 1 will never bid higher than the highest type of player 2, thus $B_1(\beta_2(1)) = 1$. By (5), we get

$$\int_{F^{-1}(\alpha_2)}^{1} \frac{v_1}{v} dF(v) + \alpha_1 = 1. \tag{7}$$

Equations (6) and (7) uniquely determine the mass points α_1 and α_2.

Lemma 4 *(i) If*

$$\int_0^1 \frac{v_1}{v} dF(v) > 1, \tag{8}$$

then $\alpha_1 = 0$ and α_2 is the unique solution to

$$\int_{F^{-1}(\alpha_2)}^{1} \frac{v_1}{v} dF(v) = 1. \tag{9}$$

(ii) If (8) does not hold, then $\alpha_2 = 0$ *and* α_1 *is the unique solution to*

$$\int_0^1 \frac{v_1}{v} dF(v) + \alpha_1 = 1. \tag{10}$$

Using Lemmas 1-4, we can now state the main result of this section.

Proposition 1 *Suppose that player* 1*'s valuation is common knowledge and player* 2*'s valuation is his private information. The all-pay auction has a unique equilibrium. Player* 1 *randomizes according to*

$$B_1(x_1) = \begin{cases} \int_0^{x_1} \frac{1}{F^{-1}\left(\frac{z+\alpha_2 v_1}{v_1}\right)} dz + \alpha_1 & \text{for} \quad x_1 \in [0, (1-\alpha_2) v_1) \\ 1 & \text{for} \quad x_1 \geq (1-\alpha_2) v_1 \end{cases} \tag{11}$$

where α_1 *and* α_2 *are defined in Lemma 4. Player* 2 *plays the following pure strategy:*

$$\beta_2(v_2) = \begin{cases} 0 & \text{for} \quad v_2 \in [0, F^{-1}(\alpha_2)) \\ F(v_2) v_1 - \alpha_2 v_1 & \text{for} \quad v_2 \in [F^{-1}(\alpha_2), 1] \end{cases} \tag{12}$$

In equilibrium, player 1 randomizes according to a (concave) distribution function. The probability that he bids zero is equal to α_1. Thus, whenever $\alpha_1 > 0$, player 1's expected payoff is zero, since he is indifferent between bidding zero and any positive bid in $(0, (1-\alpha_2) v_1]$. Player 2 bids zero for all types that are smaller than $\underline{v} = F^{-1}(\alpha_2)$, i.e. with probability α_2. For all other types, player 2 bids a positive amount $\beta_2(v_2)$ and gets a positive expected payoff which is increasing in his type. From an ex ante point of view, player 2's equilibrium payoff is strictly positive. His bid distribution is given by

$$B_2(x_2) = F\left(\beta_2^{-1}(x_2)\right) = \alpha_2 + \frac{x_2}{v_1}$$

where $x_2 \in [0, (1-\alpha_2) v_1]$. Hence, player 2's bids are uniformly distributed on $(0, (1-\alpha_2) v_1)$ with (possibly) a mass point at zero. This is similar to the all-pay auction under complete information: in order to make player 1 indifferent, player 2's bids have to follow a uniform distribution with slope $1/v_1$.

Note that, if v_1 is weakly larger than player 2's expected valuation $E(V_2)$, (8) is always fulfilled. This follows from

$$\int_0^1 \frac{v_1}{v} dF(v) \geq \int_0^1 \frac{E(V_2)}{v} dF(v) > 1 \tag{13}$$

which is true by Jensen's inequality $(E(1/V_2) > 1/E(V_2))$. Thus, if v_1 is sufficiently large, $B_2(0) > B_1(0) = 0$: player 1's willingness to bid more aggressively induces player 2 to bid zero if he has a low value.

4 An application to information acquisition

In the following, we use our results of the previous section to analyze a game of information acquisition in conflicts, focussing on the case where the decision to acquire information can be observed by the opponent, but not the acquired information itself. (We discuss the cases of open and covert information acquisition in Section 5.) As before, the players' types are independent draws from a CDF F that is common knowledge. Prior to the all-pay auction, the players simultaneously decide whether to purchase a perfectly informative signal about their own valuation at a cost c. The realization of the signal is private information, but whether or not a player has acquired information is common knowledge in the all-pay auction.

Case 1: No information acquisition. Suppose that no player acquired information. Maximizing his expected payoff in the all-pay auction, a player i's optimal strategy is to choose his effort as if his valuation were equal to his expected valuation $E(V_i) = \int_{v=0}^{v=1} v dF(v)$. Hence, the all-pay auction is reduced to a game where the (expected) valuations $E(V_1)$ and $E(V_2)$ are common knowledge. The equilibrium of the all-pay auction under complete information is in mixed strategies and is derived in Baye et al. (1996): both players randomize uniformly with support $[0, E(V_1)]$.

Fact 1 *(Baye et al. 1996) Suppose that no player acquired information. In the unique equilibrium of the all-pay auction, expected payoffs are* $E[u_1] = E[u_2] = 0$.

If no player invests in information, and both players have the same expected valuation, there is full rent dissipation in the all-pay auction.

Case 2: Two-sided asymmetric information. Suppose that both players have acquired information and know their own type, but only know the distribution of the opponent's type. In this case, the equilibrium of the all-pay auction is well-known.[4] Each player's bid is strictly increasing in his valuation.

Fact 2 *(Weber 1985, Hillman and Riley 1989) Suppose both players acquired information. In the unique equilibrium of the all-pay auction, expected payoffs are*

$$E[u_1] = E[u_2] = \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - c. \tag{14}$$

The support of the bid distributions is $[0, E(V_1)]$, as in the case without information acquisition. Without information acquisition, however, the distribution of the bids first-order stochastic dominates the bid distribution in the case of private information. Therefore, expected expenditures in the contest are lower with private information, and the players get a positive expected payoff. Moreover, the allocation of the prize is efficient in the case of private information since the player with the higher valuation wins with probability 1. Obviously, whenever c is sufficiently small, both players are better off than they are without information acquisition.

Case 3: One-sided asymmetric information. Suppose that only player 2 acquired information. Then player 2's valuation is private information, and player 1's optimal strategy is to bid as if his true valuation were $E(V_1)$. Thus, we can build on the results of Section 3 by just replacing v_1 with $E(V_1)$. Since $E(V_1) = E(V_2)$, it follows with (13) that $B_2(0) = \alpha_2 > 0$, and α_2 is defined by (9). Since player 2 bids zero for types smaller than

$$\underline{v} = F^{-1}(\alpha_2) > 0, \tag{15}$$

[4] See, for example, Krishna (2002), pp. 33-34. Uniqueness of the equilibrium follows from Amann and Leininger (1996).

the uninformed player 1 has a positive expected payoff,

$$E[u_1] = F(\underline{v})\, E(V_1) > 0. \tag{16}$$

A type $v_2 > \underline{v}$ of player 2 that bids a strictly positive amount gets a payoff of

$$B_1(\beta_2(v_2))\, v_2 - \beta_2(v_2) = \int_{\underline{v}}^{v_2} \left(\frac{E(V_1)\, v_2}{v} - E(V_1) \right) dF(v).$$

Player 2's ex ante expected payoff is therefore equal to

$$E[u_2] = \int_{\underline{v}}^{1} \int_{\underline{v}}^{v_2} \left(\frac{E(V_1)\, v_2}{v} - E(V_1) \right) dF(v)\, dF(v_2). \tag{17}$$

We now turn to the implications for the incentives to invest in information.[5]

Proposition 2 *There are two critical values $\underline{c}$ and $\bar{c}$ with $0 < \underline{c} < \bar{c}$ such that:*

(i) *If the cost of information c is strictly smaller than $\underline{c}$, both players acquire information.*

(ii) *If $\underline{c} < c < \bar{c}$, there are two equilibria where exactly one player acquires information. Additionally, there is a symmetric equilibrium where player i acquires information with probability $p = (\bar{c} - c) / (\bar{c} - \underline{c})$.*

(iii) *If $c > \bar{c}$, no player acquires information.*

The critical value $\underline{c}$ ($\bar{c}$) is the maximum amount a player is willing to spend on information given that the opponent does (does not) acquire information. It is crucial to show that $0 < \underline{c} < \bar{c}$. Since the willingness to pay for information is smaller if the opponent acquires information ($\underline{c} < \bar{c}$), an interval $(\underline{c}, \bar{c})$ exists where only one player invests in information (or both players randomize).

[5] Note that players have no private information when they decide whether to acquire information. Any reasonable belief about the opponent's type is simply the prior distribution F. Moreover, any continuation game has a unique Bayesian equilibrium. Therefore, we study the 2-by-2 game defined by the payoffs described in Facts 1-3. This amounts to studying the perfect Bayesian equilibria of the game defined in Section 2.

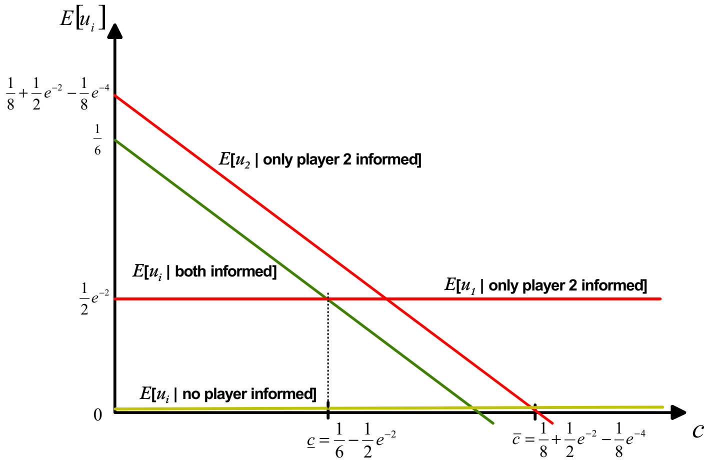

Figure 1: Payoffs dependent on c (for $F(v) = v$).

Figure 1 illustrates this result by showing the players' expected payoffs dependent on the information cost and the information decisions for uniformly distributed types $(F(v) = v)$. Obviously, for sufficiently high cost of information, no player will buy it. On the other hand, for sufficiently low cost of information, at least one player has an incentive to acquire information due to the complete rent dissipation in the case of no private information. For any continuous distribution function F, however, there is an intermediate range of information costs where it only pays for one player to acquire information.

We conclude this section by studying the efficiency of equilibrium information acquisition. We compare equilibrium information acquisition with first best investments by a social planner who is ex ante uninformed about the valuations, but can observe the outcome of any information acquisition. For concreteness, assume that the social planner derives no value from the bids in the contest and allocates the prize to the player with the higher expected valuation.

Proposition 3 *In an equilibrium without randomization concerning information acquisition, the number of players acquiring information is higher than in the first best.*

In the appendix, we show that first best investments are characterized by two

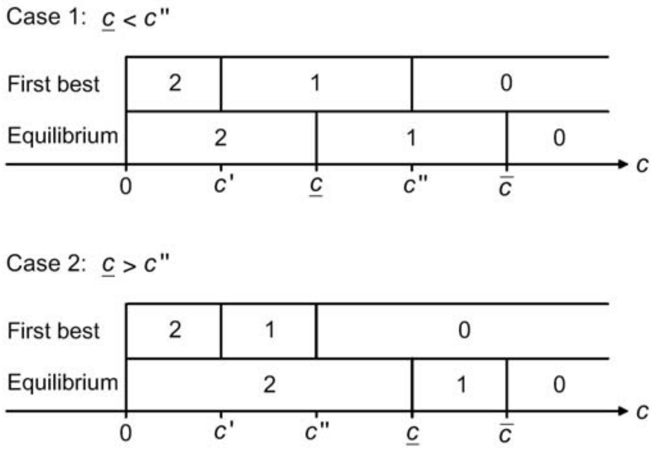

Figure 2: The number of contestants who invest in information, in the first best, and in an equilibrium without randomization of information acquisition, as a function of the cost of information acquisition c.

critical values c' and c'' (with $c' < c''$) for the cost of information: if $c < c'$, both players should invest, if $c \in (c', c'')$, one player should invest, and if $c > c''$, none should invest. The critical values that determine the social planner's investments are lower than the corresponding equilibrium thresholds of Proposition 2: $c' <\underline{c}$ and $c'' < \bar{c}$. Depending on the functional form of F, c'' can be higher or smaller than $\underline{c}$.[6] Figure 2 compares equilibrium investments with the first best. In equilibrium, there is more information acquisition: if $c \in (c', \underline{c})$, both players acquire information although only one player at most should, and similarly, whenever $c \in (c'', \bar{c})$, at least one player acquires information, though neither of the players should.[7]

[6] For example, for $F(v) = v$, $\underline{c} < c'' < \bar{c}$, and for $F(v) = v^3$, we have $c'' < \underline{c} < \bar{c}$.

[7] In the symmetric equilibrium with randomization of information acquisition, under some parameter constellations, it may happen that ex post no player invests although in the first best one player should invest. To be more precise, if $c'' \leq \underline{c}$, then the number of players acquiring information is always weakly higher than in the first best. On the other hand, if $c'' > \underline{c}$, then for any $c \in (\underline{c}, c'')$, in the first best exactly one player acquires information, whereas in the mixed equilibrium the number of players acquiring information is zero, one, or two, depending on the realizations of players' randomization.

5 Observability of information acquisition

The analysis in the previous section builds on a crucial assumption on the observability of information acquisition: we assumed that the players' decisions whether to acquire information are observable, but the information itself is only privately known to a player. In the following, we discuss this assumption by modifying it in two different directions. On the one hand, we consider the case where both the players' decisions and the information is publicly observable (open information acquisition), and on the other hand, we discuss the case where neither the information nor the players' decisions are observable by the opponent (covert information acquisition).

With open information acquisition, there are three different situations that can arise in the all-pay auction. If no player acquired information, the equilibrium is as described in Fact 1. If only player i acquired information, i's valuation is common knowledge, and j bids as if his value was $E(V_j)$. If both players acquired information, both v_i and v_j are common knowledge. In all three cases, the equilibrium is similar to the equilibrium under complete information characterized by Baye et al. (1996). Comparing the expected payoffs in the three cases determines the amount that the players are willing to spend on information.

Proposition 4 *With open information acquisition, in any equilibrium without randomization concerning information acquisition, players invest as in the first best.*

If the information that players acquire is observable, cut-off values exist for the cost of information such that both, only one, or none of the players wants to invest in information. These thresholds, however, are exactly the same as the thresholds a social planner would set (c' and c''). Thus, if the information is publicly observable, players invest less in information, and information acquisition is efficient.

Now turn to the case of covert information acquisition where a player cannot observe whether or not the other player has acquired information. Intuitively, for a very low cost of information, both players invest, and for very high cost, no player invests in information.

Proposition 5 *With covert information acquisition, (i) there is an equilibrium where both players invest in information if and only if $c < c'$, and (ii) there is an equilibrium where no player invests in information if and only if $c > c''$.*

For the sake of brevity, we do not characterize the equilibria for the entire range of cost parameters c,[8] but, interestingly, the cut-off values for c such that both players, or none of the players, acquire information are as in the first best. Thus, a player is willing to spend more on information if the decisions are observable than if the decisions are not observable by the other player.

6 Conclusion

We considered the all-pay auction between two players with one-sided asymmetric information. The asymmetry accounts for the fact that there may be superior information about one of the contestants, for example an incumbent, compared to the other contestants. We showed that if one contestant's value of winning is publicly known and the value of the opponent is private information, the all-pay auction has a unique equilibrium, and we characterized the equilibrium strategies.

Building on this result, we studied the contestants' incentives to invest in information before they compete in the all-pay auction. We distinguished between three different scenarios: (i) the opponent can observe only *that* a player has acquired information, but not *what* information he received, (ii) the opponent can observe the information itself (open information acquisition), and (iii) the opponent does not observe the decision to acquire information (covert information acquisition). In all scenarios, if the cost of information is sufficiently low, it is outweighed by the value that the information has in the contest. For intermediate cost of information, however, only one player may invest in information. Therefore, in scenario (i), in the all-pay auction one contestant may have private information whereas there are

[8] For $c \in (c', c'')$, one has to include situations where players randomize their information choice, in which case i bids against an informed player j with some probability. The equilibrium of the all-pay auction is then as if types are private information and drawn from a continuous distribution function exhibiting one interior discontinuity.

common beliefs about the other contestant's value of winning. Moreover, in equilibrium, more information is acquired than in the first best. In contrast, with open or covert information acquisition, the cut-off values for the cost of information acquisition are as in the first best. In all three scenarios, although players are symmetric ex ante, rent dissipation is incomplete unless the costs of information acquisition are prohibitive.

An interesting extension of our work could be the case of N contestants and asymmetric information. For example, if a monopolist tries to defend the monopoly rents against multiple entrants, there might be asymmetric information in the sense that one contestant's type is common knowledge and the other $(N-1)$ contestants' types are private information. The structure of the equilibrium should then be similar to the two-players case.

A Appendix

A.1 Proof of Lemma 1

(i) *(Continuity)* Suppose that B_j exhibits a discontinuity at some $\tilde{x}>0$. This implies that a bid of $x_j=\tilde{x}$ has strictly positive probability. Thus, there exist $\varepsilon,\varepsilon'>0$ such that player i strictly prefers $x_i=\tilde{x}+\varepsilon$ over all $x_i\in(\tilde{x}-\varepsilon',\tilde{x})$: shifting probability mass from $(\tilde{x}-\varepsilon',\tilde{x})$ to $\tilde{x}+\varepsilon$ only involves an infinitesimally larger cost of effort, but strictly increases the probability of winning.[9] Since player i will not bid in $(\tilde{x}-\varepsilon',\tilde{x})$, player j can strictly increase his payoff by bidding $\tilde{x}-\frac{\varepsilon'}{2}$ instead of $\tilde{x}$.

(ii) *(Support)* Let $\bar{b}_i$ ($\underline{b}_i$) denote the maximum (minimum) of the support of B_i. Suppose that $\bar{b}_i>\bar{b}_j$. Then $B_j(x)=1$ for all $x\geq\bar{b}_j$. Thus, player i prefers to bid $x_i=\left(x_i'+\bar{b}_j\right)/2$ to any bid $x_i'>\bar{b}_j$, contradicting $\bar{b}_i>\bar{b}_j$. Hence, $\bar{b}_1=\bar{b}_2=\bar{b}$. Since player 1 can ensure a payoff of zero by bidding zero, we must have $\bar{b}\leq v_1$.

Suppose that $\underline{b}_i>\underline{b}_j>0$. Then any bid $x_j<\underline{b}_i$ loses with probability one; player j could increase his payoff by bidding zero instead, which is a contradiction.

[9] If $i=2$, this argument assumes $v_2>0$. But this is inconsequential since type $v_2=0$ has zero probability.

Now suppose $\underline{b}_i > \underline{b}_j = 0$. Then player j strictly prefers a bid of zero over all bids in $(0, \underline{b}_i)$, thus B_j has no probability mass in $(0, \underline{b}_i)$. Since B_j has no mass points (except possibly at zero) it follows that B_j is constant on $(0, \underline{b}_i]$. But then there exists $\varepsilon > 0$ such that player i strictly prefers a bid of ε over any bid in $[\underline{b}_i, \underline{b}_i + \varepsilon)$: a bid of ε has strictly lower costs but only a marginally lower probability of winning. This is a contradiction to the definition of $\underline{b}_i$.

Finally, suppose $\underline{b}_1 = \underline{b}_2 = \underline{b} > 0$. By (i), $B_j(\underline{b}) = 0$, and there exists an $\varepsilon > 0$ such that $x_i = 0$ is preferred to any bid $x_i \in [\underline{b}, \underline{b} + \varepsilon)$, which contradicts $\underline{b}_i > 0$. Combining these arguments shows that $\underline{b}_1 = \underline{b}_2 = 0$.

(iii) *(Mass points at zero)* If $B_j(0) > 0$, there exists an $\varepsilon > 0$ such that player i prefers $x_i = \varepsilon$ to $x_i = 0$. Hence, $B_i(0) = 0$. This shows that the bid distribution of at most one player can have a mass point at zero.

(iv) *(Monotonicity)* Suppose that B_j is constant in an interval (x', x'') where $0 \leq x' < x'' \leq \bar{b}$, further suppose that $x'' = \max\{x \,| B_j(x) = B_j(x')\}$. Then $B_j(x') = B_j(x'') < 1$ since $x' < \bar{b}$. There exists an $\varepsilon > 0$ such that player i prefers $x_i = x'$ to all $x_i \in (x', x'' + \varepsilon)$: by bidding x' player i reduces his probability of winning only by (at most) an infinitesimally small amount, but strictly decreases his expected cost of effort. Thus i does not bid in $(x', x'' + \varepsilon)$. Since B_i has no mass points, we have $B_i(x') = B_i(x'' + \varepsilon)$. But then j prefers bidding x' over any bid in $[x'', x'' + \varepsilon]$ and thus we must have $B_j(x'' + \varepsilon) = B_j(x')$, contradicting $x'' = \max\{x \,| B_j(x) = B_j(x')\}$.

A.2 Proof of Lemma 2

First we show that no type of player 2 randomizes. Suppose to the contrary that some type v_2' of player 2 does randomize. Let c_l (c_h) be the infimum (supremum) of the support of the distribution of bids made by type v_2'. For any $c > c_l$,

$$B_1(c_l)\, v_2' - c_l \geq B_1(c)\, v_2' - c \tag{18}$$

for otherwise v_2' could gain from shifting probability mass to c.[10] From (18),

$$c - c_l \geq (B_1(c) - B_1(c_l))\, v_2'.$$

Since B_1 is strictly increasing, for any $v_2'' < v_2'$ we have

$$c - c_l > (B_1(c) - B_1(c_l))\, v_2''$$

or

$$B_1(c_l)\, v_2'' - c_l > B_1(c)\, v_2'' - c$$

i.e. type v_2'' strictly prefers to bid c_l over bidding c. Therefore, for all $v_2'' < v_2'$, the supremum of the support of the distribution of bids made by type v_2'' must be weakly smaller than c_l. Similarly, for all $v_2''' > v_2'$, the infimum of the support of the distribution of bids made by type v_2''' must be weakly higher than c_h. Therefore only type v_2' bids in (c_l, c_h). Since the distribution of types, F, is continuous, it follows that B_2 is constant on (c_l, c_h), contradicting Lemma 1.

It follows that player 2 plays a pure strategy $\beta_2 : [0,1] \to [0, \infty)$. Moreover, β_2 is weakly increasing. Now suppose that $v_2' < v_2''$ and $\beta_2(v_2') = \beta_2(v_2'')$. Since β_2 is weakly increasing, it follows that $\beta_2(v_2) = \beta_2(v_2')$ for all $v_2 \in [v_2', v_2'']$. Therefore B_2 has an atom at $\beta_2(v_2')$ (the size of the atom is at least $F(v_2'') - F(v_2')$). Since B_2 is continuous except possibly at zero, this atom can only be at $\beta_2(v_2') = 0$.

This shows that there is a $\underline{v} \in [0,1)$ such that, first, for all $v_2 \leq \underline{v}$, $\beta_2(v_2) = 0$, and second, β_2 is strictly increasing on $[\underline{v}, 1]$. Since B_2 is strictly increasing, β_2 has to be continuous as well.

[10] If c_l has strictly positive probability, type v_2' gains from shifting this probability mass to c. If c_l has zero probability, then, for any $\varepsilon > 0$, the interval $(c_l, c_l + \varepsilon)$ has positive probability. By continuity of B_1, if (18) does not hold, then for small enough $\varepsilon > 0$, $B_1(c_l + \varepsilon)\, v_2' - (c_l + \varepsilon) < B_1(c)\, v_2' - c$, and shifting probability mass from the interval $(c_l, c_l + \varepsilon)$ to c is beneficial.

A.3 Proof of Lemma 3

We first show that B_1 is differentiable at any $x_2 \in \left(0, \bar{b}\right)$. Let $v_2 = \beta_2^{-1}\left(x_2\right)$ and consider a strictly increasing sequence v_2^n with $v_2^n \in \left(\underline{v}, 1\right)$ and $\lim_{n\to\infty} v_2^n = v_2$. For notational brevity let $x_2^n = \beta_2\left(v_2^n\right)$. Then x_2^n is strictly increasing and $\lim_{n\to\infty} x_2^n = x_2$.

Bidding x_2^n is at least as good as bidding x_2 for type v_2^n, thus

$$B_1\left(x_2^n\right) v_2^n - x_2^n \geq B_1\left(x_2\right) v_2^n - x_2$$

or

$$1 \geq v_2^n \frac{B_1\left(x_2\right) - B_1\left(x_2^n\right)}{x_2 - x_2^n}.$$

Taking lim sup, we get

$$\limsup \left(\frac{B_1\left(x_2\right) - B_1\left(x_2^n\right)}{x_2 - x_2^n}\right) \leq \frac{1}{v_2}. \tag{19}$$

Similarly, for type v_2, bidding x_2 is at least as good as bidding x_2^n. Thus

$$B_1\left(x_2\right) v_2 - x_2 \geq B_1\left(x_2^n\right) v_2 - x_2^n.$$

Rearranging and taking lim inf, we get

$$\liminf \left(\frac{B_1\left(x_2\right) - B_1\left(x_2^n\right)}{x_2 - x_2^n}\right) \geq \frac{1}{v_2}. \tag{20}$$

From (20) and (19), it follows that

$$\lim_{x_2^n \uparrow x_2} \left(\frac{B_1\left(x_2\right) - B_1\left(x_2^n\right)}{x_2 - x_2^n}\right) = \frac{1}{v_2}.$$

A parallel argument, that considers a strictly decreasing sequence v_2^n with limit v_2, shows that

$$\lim_{x_2^n \downarrow x_2} \left(\frac{B_1\left(x_2\right) - B_1\left(x_2^n\right)}{x_2 - x_2^n}\right) = \frac{1}{v_2}.$$

Thus B_1 is differentiable at v_2, with

$$\left.\frac{dB_1(x)}{dx}\right|_{x=x_2} = \lim_{x_2^n \to x_2} \left(\frac{B_1(x_2) - B_1(x_2^n)}{x_2 - x_2^n} \right) = \frac{1}{v_2}.$$

We next show that the bid distribution B_2 is differentiable. Since B_1 is strictly increasing on $(0, \bar{b})$, player 1 must be indifferent between all bids $x \in (0, \bar{b})$. Fix one $x_1 \in (0, \bar{b})$. Consider a sequence x_1^n with limit x_1 and with $x_1^n \in (0, \bar{b})$ for all n. For all n, player 1 is indifferent between bidding x_1^n and bidding x_1 :

$$B_2(x_1^n) v_1 - x_1^n = B_2(x_1) v_1 - x_1$$

Rearranging,

$$\frac{B_2(x_1) - B_2(x_1^n)}{x_1 - x_1^n} = \frac{1}{v_1}$$

Thus

$$\lim_{n \to \infty} \left(\frac{B_2(x_1) - B_2(x_1^n)}{x_1 - x_1^n} \right) = \frac{1}{v_1}$$

and therefore B_2 is differentiable.

Since F is differentiable by assumption, it follows that β_2 must be differentiable as well.

A.4 Proof of Lemma 4

(i) Suppose to the contrary that $\alpha_1 > 0$. Then $\alpha_2 = 0$ by (6) and thus

$$B_1(\beta_2(1)) = \int_0^1 \frac{v_1}{v} dF(v) + \alpha_1 > 1,$$

contradiction. Thus $\alpha_1 = 0$. Inserting $\alpha_1 = 0$ in (7), we get (9). The left-hand side of (9) is strictly greater than one for $\alpha_2 = 0$, it strictly decreases in α_2, and is equal to zero for $\alpha_2 = 1$. By continuity, there is a unique $\alpha_2 \in (0, 1)$ such that (9) holds. Part (ii) can be proven similarly. From (i) and (ii), it follows that α_1 and α_2 are uniquely determined.

A.5 Proof of Proposition 1

Uniqueness follows from the discussion in the main text. It remains to establish that the strategies are an equilibrium. Consider player 1 and suppose player 2 follows (12). The expected payoff of player 1 for a bid $x_1 \in (0, (1 - \alpha_2) v_1]$ is equal to

$$E [u_1 (x_1)] = F \left(\beta_2^{-1} (x_1)\right) v_1 - x_1$$

since β_2^{-1} exists on $(0, (1 - \alpha_2) v_1]$. Inserting (12), we get $E [u_1 (x_1)] = \alpha_2 v_1$ for all $x_1 \in (0, (1 - \alpha_2) v_1]$. Moreover, if (8) does not hold, then $\alpha_2 = 0$ and player 1 has a payoff of zero; thus in this case he is indifferent between all $x_1 \in [0, (1 - \alpha_2) v_1]$. Bidding more than $(1 - \alpha_2) v_1$ is always suboptimal. Thus (11) is a best response.

Now consider player 2 and suppose he has a valuation v_2. Given B_1, his payoff $B_1 (x) v_2 - x$ is strictly concave in his bid x since

$$B_1'' (x) = \frac{\partial^2}{\partial x^2} \left(\int_0^x \frac{1}{F^{-1}\left(\frac{z+\alpha_2 v_1}{v_1}\right)} dz \right) = \frac{\partial}{\partial x} \frac{1}{F^{-1}\left(\frac{x+\alpha_2 v_1}{v_1}\right)} < 0.$$

If $v_2 > F^{-1} (\alpha_2)$, then the first-order condition (3) describes the unique maximum. If $v_2 \leq F^{-1} (\alpha_2)$, then for all $x_2 > 0$,

$$B_1' (x_2) v_2 - 1 = \frac{1}{F^{-1}\left(\frac{x_2+\alpha_2 v_1}{v_1}\right)} v_2 - 1 < 0.$$

Therefore, (12) is a best response.

A.6 Proof of Proposition 2

Suppose player j does not acquire information. If i does not acquire information either, he gets an expected payoff of zero by Fact 1; if i acquires information, his payoff is described by (17). Hence, i's best response is to acquire information if and only if c is smaller than

$$\bar{c} := \int_{\underline{v}}^{1} \int_{\underline{v}}^{v_i} \left(\frac{E(V)}{v_j} v_i - E(V) \right) dF(v_j)\, dF(v_i) \tag{21}$$

where, from (15), $\underline{v} = F^{-1}(\alpha_i) > 0$, and $\underline{v}$ is defined by

$$\int_{\underline{v}}^{1} \frac{E(V)}{v} dF(v) = 1. \tag{22}$$

Note that from (22), it follows that $\underline{v} < E(V)$.

Now suppose that j acquires information. If i remains uninformed, he gets $F(\underline{v})\, E(V)$, as in (16). If i acquires information, his payoff is described by (14). Thus, i's best response is to acquire information if and only if c is smaller than

$$\underline{c} := \int_{0}^{1} \int_{0}^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_{0}^{\underline{v}} E(V)\, dF(v) \tag{23}$$

where again $\underline{v}$ is defined by (22).

Let

$$c' := E_{v_i, v_j}\left[\max\{v_i, v_j\}\right] - E_{v_j}\left[\max\{E(V), v_j\}\right].$$

(In Appendix A.7, we will show that in the first best, both players acquire information if and only if $c < c'$.) The following lemmas will be used repeatedly below.

Lemma 5

$$c' = \int_{0}^{1} \int_{0}^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_{0}^{E(V)} (E(V) - v_j)\, dF(v_j) > 0.$$

Proof. For the equality,

$$
\begin{aligned}
c' &= E_{v_i,v_j}\left[\max\left\{v_i,v_j\right\}\right]-E_{v_j}\left[\max\left\{E\left(V\right),v_j\right\}\right] \\
&= \int_0^1\int_0^{v_i} v_i dF\left(v_j\right)dF\left(v_i\right)+\int_0^1\int_{v_i}^1 v_j dF\left(v_j\right)dF\left(v_i\right) \\
&\quad -\int_0^{E(V)} E\left(V\right)dF\left(v_j\right)-\int_{E(V)}^1 v_j dF\left(v_j\right) \\
&= \int_0^1\int_0^{v_i}\left(v_i-v_j\right)dF\left(v_j\right)dF\left(v_i\right)-\int_0^{E(V)}\left(E\left(V\right)-v_j\right)dF\left(v_j\right).
\end{aligned}
$$

The inequality $c'>0$ follows from Jensen's inequality. To see this, define

$$
g\left(v_i\right):=\int_0^1 \max\left\{v_i,v_j\right\}dF\left(v_j\right).
$$

Since g is strictly convex in v_i,

$$
E_{v_i}\left[g\left(v_i\right)\right]>g\left(E_{v_i}\left(v_i\right)\right)
$$

or equivalently

$$
E_{v_i,v_j}\left[\max\left\{v_i,v_j\right\}\right]>E_{v_j}\left[\max\left\{E\left(V\right),v_j\right\}\right].
$$

■

Lemma 6 *(i)* $\underline{c}> c'$ *and (ii)* $\bar{c}>\underline{c}$.

Proof. (i) Using Lemma 5,

$$
\begin{aligned}
\underline{c}-c' &= \int_0^{E(V)}\left(E\left(V\right)-v_j\right)dF\left(v_j\right)-\int_0^{\underline{v}} E\left(V\right)dF\left(v_j\right) \\
&= \int_{\underline{v}}^{E(V)}\left(E\left(V\right)-v_j\right)dF\left(v_j\right)-\int_0^{\underline{v}} v_j dF\left(v_j\right).
\end{aligned}
$$

Adding and subtracting both $\int_0^{E(V)} \underline{\mathrm{v}} dF(v_j)$ and $\int_{\underline{\mathrm{v}}}^{E(V)} \underline{\mathrm{v}} \frac{E(V)}{v_j} dF(v_j)$ yields

$$\begin{aligned} \underline{c} - c' &= \int_0^{\underline{\mathrm{v}}} (\underline{\mathrm{v}} - v_j)\, dF(v_j) + \int_{\underline{\mathrm{v}}}^{E(V)} \left(E(V) - v_j + \underline{\mathrm{v}} - \underline{\mathrm{v}} \frac{E(V)}{v_j} \right) dF(v_j) \\ &\quad - \int_0^{E(V)} \underline{\mathrm{v}} dF(v_j) + \int_{\underline{\mathrm{v}}}^{E(V)} \underline{\mathrm{v}} \frac{E(V)}{v_j} dF(v_j). \end{aligned}$$

First observe that

$$\begin{aligned} \int_{\underline{\mathrm{v}}}^{E(V)} \underline{\mathrm{v}} \frac{E(V)}{v_j} dF(v_j) &= \underline{\mathrm{v}} \left[\int_{\underline{\mathrm{v}}}^{1} \frac{E(V)}{v_j} dF(v_j) - \int_{E(V)}^{1} \frac{E(V)}{v_j} dF(v_j) \right] \\ &= \underline{\mathrm{v}} \left[1 - \int_{E(V)}^{1} \frac{E(V)}{v_j} dF(v_j) \right] \end{aligned}$$

where the second equality uses (22). Therefore,

$$\begin{aligned} \underline{c} - c' &= \int_0^{\underline{\mathrm{v}}} (\underline{\mathrm{v}} - v_j)\, dF(v_j) + \int_{\underline{\mathrm{v}}}^{E(V)} \frac{(E(V) - v_j)(v_j - \underline{\mathrm{v}})}{v_j} dF(v_j) \\ &\quad + \underline{\mathrm{v}} \left[1 - \int_0^{E(V)} dF(v_j) - \int_{E(V)}^{1} \frac{E(V)}{v_j} dF(v_j) \right] \end{aligned}$$

which is strictly positive.

(ii) With (21) and (23), $\bar{c} - \underline{c}$ is equal to

$$\begin{aligned} &\int_{\underline{\mathrm{v}}}^{1} \int_{\underline{\mathrm{v}}}^{v_i} \left(\frac{E(V)\, v_i}{v_j} - E(V) \right) dF(v_j)\, dF(v_i) \\ &\qquad + \int_0^1 \int_0^{\underline{\mathrm{v}}} E(V)\, dF(v_j)\, dF(v_i) - \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) \\ &= \int_0^1 h(v_i)\, dF(v_i) \end{aligned}$$

where

$$h(v_i) = \int_0^{\underline{\mathrm{v}}} E(V)\, dF(v_j) - \int_0^{v_i} (v_i - v_j)\, dF(v_j)$$

if $v_i \leq \underline{v}$, and

$$\begin{aligned} h(v_i) &= \int_{\underline{v}}^{v_i} \left(\frac{E(V) v_i}{v_j} - E(V) \right) dF(v_j) \\ &\quad + \int_0^{\underline{v}} E(V) dF(v_j) - \int_0^{v_i} (v_i - v_j) dF(v_j) \end{aligned}$$

if $v_i > \underline{v}$. Then, it is sufficient to show that $h(v_i) > 0$ for all $v_i \in [0,1]$.
Case 1: $v_i \leq \underline{v}$. From (22), it follows that $\underline{v} < E(V)$, and thus

$$\int_0^{\underline{v}} E(V) dF(v_j) > \int_0^{v_i} v_i dF(v_j) > \int_0^{v_i} (v_i - v_j) dF(v_j).$$

Case 2: $v_i \in (\underline{v}, E(V)]$. Here, $h(v_i)$ is equal to

$$\int_{\underline{v}}^{v_i} \frac{(v_i - v_j)(E(V) - v_j)}{v_j} dF(v_j) + \int_0^{\underline{v}} (E(V) - v_i + v_j) dF(v_j).$$

The first term is strictly positive because $v_j \leq v_i \leq E(V)$ and $v_i > \underline{v}$. The second term is strictly positive as $v_i \leq E(V)$ and, by (15), $\underline{v} > 0$.
Case 3: $v_i \in (E(V), 1]$. Since $\underline{v}$ is independent of v_i, we get

$$\begin{aligned} h'(v_i) &= \int_{\underline{v}}^{v_i} \frac{E(V)}{v_j} dF(v_j) - \int_0^{v_i} dF(v_j), \\ h''(v_i) &= \frac{E(V)}{v_i} F'(v_i) - F'(v_i), \end{aligned}$$

hence, h is strictly concave for $v_i > E(V)$. Moreover, as $v_i \to 1$, h' converges to

$$\int_{\underline{v}}^{1} \frac{E(V)}{v_j} dF(v_j) - \int_0^1 dF(s_j) = 1 - 1 = 0.$$

(The first integral is one by (22).) Thus, h' must be positive for all $v_i \in (E(V), 1)$ and thus $h(v_i) > h(E(V)) > 0$ where the last inequality follows from case 2. ■

We are now in a position to prove Proposition 2. From Lemmas 5 and 6, it follows directly that $\bar{c} > \underline{c} > 0$. Thus, (i) if $c < \underline{c}$, information acquisition is strictly dominant. (ii) If $\underline{c}< c < \bar{c}$, a player invests in information only if the opponent remains uninformed, and there exist two asymmetric equilibria where exactly one player invests. Moreover, there is a symmetric equilibrium where both players invest in information with probability $p = (\bar{c} - c) / (\bar{c} - \underline{c})$: if player i acquires information, he gets

$$(1 - p)(\bar{c} - c) + p(F(\underline{v})E(V) + \underline{c} - c) = pF(\underline{v})E(V)$$

which is equal to his payoff if he remains uninformed. Thus, i is indifferent between investing and not investing in information. Moreover, for all p that are strictly smaller (greater) than this critical value, i strictly prefers (not) to acquire information. Finally, (iii) if $c > \bar{c}$, not investing is strictly dominant.

A.7 Proof of Proposition 3

Consider the decision of the social planner. If she does not acquire any information, she gives the prize to any player and realizes a welfare of $E(V)$. If she acquires information about the valuation of one player, it is optimal to give the prize to this player if and only if his valuation is higher than $E(V)$. In this case, welfare is equal to $E_{v_j}[\max\{E(V), v_j\}] - c$. If the social planner acquires information about both players, welfare equals $E_{v_i,v_j}[\max\{v_i, v_j\}] - 2c$. As above, let

$$c' = E_{v_i,v_j}[\max\{v_i, v_j\}] - E_{v_j}[\max\{E(V), v_j\}]. \quad (24)$$

Moreover, let

$$\begin{aligned} c'' &= E_{v_i}[\max\{E(V), v_i\}] - E(V) \\ &= \int_{E(V)}^{1} (v_i - E(V))\, dF(v_i). \end{aligned} \quad (25)$$

If the cost of information acquisition equals c', welfare is the same if two players acquire information as if one acquires information. At c'', welfare is the same if one player acquires information as if no one does.

Lemma 7 *(i)* $0 < c' < c''$ *and (ii)* $c' <\underline{c}$ *and* $c'' < \bar{c}$.

Proof. **(i)** In Lemma 5, we have already shown that $c' > 0$. Moreover, using Lemma 5,

$$\begin{aligned}
c' &= \int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{E(V)} (E(V) - v_j)\, dF(v_j) \\
&= \int_0^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) + \int_0^1 \int_0^{v_i} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \\
&\quad - \int_0^1 \int_0^{E(V)} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \\
&= \int_0^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) + \int_{E(V)}^1 \int_{E(V)}^{v_i} (E(V) - v_j)\, dF(v_j)\, dF(v_i) \\
&\quad - \int_0^{E(V)} \int_{v_i}^{E(V)} (E(V) - v_j)\, dF(v_j)\, dF(v_i)
\end{aligned}$$

which is strictly smaller than

$$\begin{aligned}
\int_0^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) &< \int_{E(V)}^1 \int_0^{v_i} (v_i - E(V))\, dF(v_j)\, dF(v_i) \\
&< \int_{E(V)}^1 \int_0^1 (v_i - E(V))\, dF(v_j)\, dF(v_i) = c''.
\end{aligned}$$

(ii) The first inequality is Lemma 6, part (i). Moreover, by (21) and (25), $\bar{c} > c''$ is equivalent to

$$\int_{\underline{v}}^1 \int_{\underline{v}}^{v_i} \left(\frac{E(V)\, v_i}{v_j} - E(V) \right) dF(v_j)\, dF(v_i) > \int_{E(V)}^1 (v_i - E(V))\, dF(v_i).$$

By (17), the left-hand side is i's ex ante expected payoff if i acquired information and j remained uninformed. Since, in this case, j never bids higher than his expected value, the LHS must be weakly higher than the RHS, because the latter is the payoff i could ensure by bidding $E(V)$ for all types $v_i \geq E(V)$ and bidding zero otherwise. It remains to show that for some realizations of v_i, i can do strictly better. Note first that $F^{-1}(\alpha_i) = \underline{v} > 0$, i.e. j's maximum bid is $\bar{b} = (1 - \alpha_i) E(V) < E(V)$. Hence, for all realizations $v_i \in ((1 - \alpha_i) E(V), E(V))$, i can ensure a strictly positive payoff by bidding $(1-\alpha_i) E(V)$, and hence the LHS must be strictly larger than the RHS.

■

The inequalities in (i) allow us to characterize first best information acquisition: if $c < c'$, both should acquire information; if $c \in (c', c'')$, exactly one player should acquire information; finally, if $c > c''$, no one should. With (ii), we can compare equilibrium investments and first best investments (see Figure 2 in the main text). If $c < c'$, both players invest as in the first best. If $c \in (c', \min\{\underline{c}, c''\})$, both players acquire information although exactly one player should. If $c \in (\min\{\underline{c}, c''\}, c'')$, in the asymmetric equilibria exactly one player acquires information, as in the first best. If $c \in (c'', \bar{c})$, at least one player acquires information, but neither of the players should. Finally, if $c > \bar{c}$, no player invests, as in the first best. Therefore, the number of players investing in information is higher than the first best.

A.8 Proof of Proposition 4

If no player invests in information, both get an expected payoff of zero. If only player i invests, i's expected payoff is $E_{v_i}[\max\{v_i - E(V), 0\}] - c$, while j gets $E_{v_i}[\max\{E(V) - v_i, 0\}]$. If both players acquire information, each of them gets $E_{v_i,v_j}[\max\{v_i - v_j, 0\}] - c$.

Now suppose that j remains uninformed. Player i's best response is to acquire information whenever c is smaller than $E_{v_i}[\max\{v_i - E(V), 0\}]$ which, with (25), is equal to c''. If j acquires information, i invests whenever c is smaller than

$$E_{v_i,v_j}[\max\{v_i - v_j, 0\}] - E_{v_i}[\max\{E(V) - v_i, 0\}]$$

which, by Lemma 5, is equal to c'. Since $0 < c' < c''$, both players (no player) acquire information if $c < c'$ $(c > c'')$. If $c \in (c', c'')$, there are two equilibria where exactly one player acquires information, and a mixed strategy equilibrium where players acquire information with probability $(c'' - c) / (c'' - c')$.

A.9 Proof of Proposition 5

(i) We first analyze whether there can be an equilibrium where both players acquire information with probability 1. If this is the case, then they bid as in Fact 2 and both get a payoff of

$$\int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - c.$$

Now suppose that i deviates and remains uninformed. Then, his optimal bid is as if he had a value of $E(V)$ which leads to a deviation payoff of

$$\int_0^{E(V)} (E(V) - v_j)\, dF(v_j).$$

Hence, it pays off to save the cost of information whenever c is larger than

$$\int_0^1 \int_0^{v_i} (v_i - v_j)\, dF(v_j)\, dF(v_i) - \int_0^{E(V)} (E(V) - v_j)\, dF(v_j)$$

which, by Lemma 5, is equal to c'. Thus, if and only if $c < c'$, an equilibrium exists where both players acquire information.

(ii) Now suppose that both players do not invest in information with probability 1. Then, both get zero payoff. If i deviates and acquires information, his optimal bid is zero if $v_i \leq E(V)$ and $E(V)$ if $v_i > E(V)$. (The type $v_i = E(V)$ is exactly indifferent. Thus, lower types prefer a bid of zero, and higher types prefer a bid at the upper bound of the support of j's bids.) The deviation payoff is

$$\int_{E(V)}^1 (v_i - E(V))\, dF(v_i) - c.$$

Therefore, if and only if c is larger than c'' (from (25)), there is an equilibrium where no player acquires information.

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